On the regulators with random noises in dynamic block

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Abstract

The problem of controlling stochastic linear systems with quadratic criterion is considered. A class of optimal controllers which are equivalent to the separation theorem regulator is determined. For all of such controllers the quadratic functional has the same value. The effects of disregarded disturbances which are modeled by random noises in the dynamic block of the regulator are investigated. It is shown that the equivalent (in the classic propounding) controllers respond to these noises in different ways. Sometimes an "equivalent optimal" regulator may be less receptive towards additional disturbances than the standard one (which comes from the separation theorem). The optimal regulator is found which takes into account the presence of such noises.

Key words: Stochastic systems, optimal regulators, separation theorem

1. Introduction

This work deals with the problem of finding an optimal feedback control for stochastic continuous-time system under incomplete information. In the case of Linear-Quadratic-Gaussian assumptions, the standard solution is given by the separation theorem [1] via two separate problems: estimation and control. Here we consider other regulators of the same structure (dynamic block+ feedback) which minimize the quadratic functional but don't satisfy the separation principle.

The note is structured as follows. In Section 2, an equivalent controller conception is introduced and the constructive description of such controller class is given. The solution of only one additional matrix Riccati equation is required to design an equivalent regulator. Since the quadratic criterion has the same value for all class determined, the separation theorem gives us only one of the possible presentations of the optimal control.

In practice, functioning of an optimal regulator is inevitably accompanied by different distortions connected with analogous or digital simulation of the dynamic block. For the estimation problem, a simple method was suggested to take account of such misrepresentations [2],[3]. Errors of all sorts were modeled by random noises in dynamic block of the observer. The present note deals with similar regulator investigations and goes along with numerous works on robust control [4],[5]. In Section 3, we consider the influence of additional noises on the "optimal" controllers. The equivalent (in the classic propounding) regulators turned out to respond to these disturbances in different ways.

Section 4 is devoted to an optimal regulator which takes into account the presence of such disturbances. The equations for parameters of the optimal regulator are derived.

Section 5 contains the detailed examination of time-invariant system. It is shown that the design of an equivalent stationary controller comes to a solution of quadratic matrix equation. The conditions of the existence of only two solutions (the separation theorem regulator and the alternative one) are presented. An example demonstrates that the alternative controller may be much better than the standard regulator in case of additional noises.

2. Equivalent Regulators

Consider a linear stochastic system described by the state equation

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + \xi(t), \ t \in [0, T] \tag{2.1}$$

and measurement equation

$$\dot{y}(t) = C(t)x(t) + \eta(t) \tag{2.2}$$

where x is a n vector, u is a r vector, y is a m vector; $A(t)$, $B(t)$ and $C(t)$ are continuously time-varying matrices of appropriate dimensions. The noise processes $\{\xi(t)\}$ and $\{\eta(t)\}$ are white Gaussian with properties

$$E\{\xi(t)\} = 0, \ E\{\xi(t)\xi(\tau)^T\} = S(t)\delta(t-\tau), \ S(t) \geq 0$$

$$E\{\eta(t)\} = 0, \ E\{\eta(t)\eta(\tau)^T\} = V(t)\delta(t-\tau), \ V(t) \geq 0$$

We assume that $x(0) = x_0$ is also Gaussian with mean and covariance given by

$$Ex_0 = m_0, \ E(x_0 - m_0)(x_0 - m_0)^T = \Delta_0.$$

Furthermore, $x_{0,}$ $\{\xi(t)\}$, $\{\eta(t)\}$are independent.

A regulator of the following structure is commonly used to control the system (2.1) dynamic block

$$\dot{z}(t) = A(t)z(t) + B(t)u(t) + F(t)(\dot{y}(t) - C(t)z(t)), \tag{2.3}$$

with initial point

$$z(0) = z_0$$

and feedback

$$u(t) = -F(t)z(t). \tag{2.4}$$

So, the state vector $z(t)$ of system (2.3) and control signal $u(t)$ from (2.4) depend only on the set of parameters $U = (z_0, F(t), K(t))$.

The cost performance for controlling is quadratic

$$J[u] = E\int_0^T [x^T(t)M(t)x(t) + u(t)^T N(t)u(t)]dt + x^T(T)Gx(T) \qquad (2.5)$$

where $M(t) \geq 0$ and $N(t) > 0$ are continuously time-varying symmetrical matrices, $G \geq 0$ is a constant matrix, $G^T = G$.

Prove the following result for regulators $U = (z_0, F(t), K(t))$ and $\tilde{U} = (\tilde{z}_0, \tilde{F}(t), \tilde{K}(t))$.

Theorem 1: Let $\Phi(t)$ be a non-singular matrix for all $t \in [0, T]$ that satisfies the differential equation

$$\dot{\Phi} = A(t)\Phi(t) + \Phi(t)(F(t)C(t) + B(t)K(t) - A(t)) - B(t)K(t) - \qquad (2.6)$$

$$-\Phi(t)F(t)C(t)\Phi(t)$$

with initial condition

$$\Phi(0) = \Phi_0$$

and the parameters of $\tilde{U}$ be given by

$$\tilde{z}_0 = \Phi_0 z_0, \;\; \tilde{F}(t) = \Phi(t)F(t), \;\; \tilde{K}(t) = K(t)\Phi^{-1}(t). \qquad (2.7)$$

Then the control signals $u(t)$ and $\tilde{u}(t)$ that are formed by U and $\tilde{U}$ are identical for all $t \in [0, T] : u(t) \equiv \tilde{u}(t)$.

Proof: Show that the state vector $z(t)$ and $\tilde{z}(t)$ of regulators U and $\tilde{U}$ are connected by $\tilde{z}(t) \equiv \Phi(t)z(t)$. The following expressions directly come from the relations (2.3),(2.6),(2.7)

$$\frac{d}{dt}(\Phi(t)z(t)) = \dot{\Phi}(t)z(t) + \Phi(t)\dot{z}(t) =$$

$$= A(t)\Phi(t)z(t) + \Phi(t)(F(t)C(t) + B(t)K(t) - A(t))z(t) - B(t)K(t)z(t) -$$

$$-\Phi(t)F(t)C(t)\Phi(t)z(t) + \Phi(t)A(t)z(t) - \Phi(t)B(t)K(t)z(t) +$$

$$+\Phi(t)F(t)(y(t) - C(t)\Phi(t)z(t)) =$$

$$= (A(t) - B(t)\tilde{K}(t))\Phi(t)z(t) + \tilde{F}(t)(y(t) - C(t)\Phi(t)z(t)),$$

$$\Phi(0)z(0) = \Phi_0 z_0$$

Since $\tilde{z}(t)$ satisfies the differential equation

$$\dot{\tilde{z}}(t) = A(t)\tilde{z}(t) - B(t)\tilde{K}(t)\tilde{z}(t) + \tilde{F}(t)(\dot{y}(t) - C(t)\tilde{z}(t)),$$

with initial condition

$$\tilde{z}(0) = \Phi_0 z_0$$

and with respect to the uniqueness of the Cauchy problem solution, we get the necessary identity

$$\tilde{z}(t) \equiv \Phi(t)z(t), \;\; t \in [0, T]$$

The following relation that comes from (2.7),

$$\tilde{u}(t) = -\tilde{K}(t)\tilde{z}(t) = -K(t)\Phi^{-1}(t)\Phi(t)z(t) \equiv -K(t)z(t) = u(t),$$

completes the theorem proof.

Note 1. If $\tilde{U}$ is given by (2.6),(2.7) then functional (2.5) has the same value for the regulators U and $\tilde{U}$: $J[u] = J[\tilde{u}]$. Therefore , *Theorem* 1 gives us a method to generate a class of equivalent regulators for any starting regulator $U = (z_0, F(t), K(t))$. Actually , we need to solve Riccati equation (2.6) with fixed initial matrix Φ_0 to define the parameters of an equivalent controller by formulas (2.7). So the variety of the equivalent regulators depends on the amount of such non-singular matrices Φ_0 for which matrices $\Phi(t)$ (the corresponding solution of (2.6)) are also non-singular for all $t \in [0,T]$.

Note 2. Let matrix $\Psi(t) = \Phi(t) - I_n$ be also non-singular for all $t \in [0,T]$. From (2.6) it is follows the next equalities

$$\dot{\Psi}(t) = A(t)\Psi(t) + A(t) + \Psi(t)(F(t)C(t) + B(t)K(t) - A(t)) +$$
$$+F(t)C(t) + B(t)K(t) - A(t) - B(t)K(t) - \Psi(t)F(t)C(t)\Psi(t) -$$
$$-\Psi(t)F(t)C(t) - F(t)C(t)\Psi(t) - F(t)C(t) =$$
$$= (A(t) - F(t)C(t))\Psi(t) + \Psi(t)(B(t)K(t) - A(t)) - \Psi(t)F(t)C(t)\Psi(t)$$

From the other side

$$\frac{d}{dt}(\Psi^{-1}(t)) = \frac{d}{dt}(\Psi^{-1}(t)\Psi(t)\Psi^{-1}(t)) = \frac{d}{dt}(\Psi^{-1}(t)) + \Psi^{-1}(t)\frac{d}{dt}(\Psi(t))\Psi^{-1}(t) + \frac{d}{dt}(\Psi^{-1}(t))$$

That is

$$\frac{d}{dt}(\Psi^{-1}(t)) = -\Psi^{-1}(t)\frac{d}{dt}(\Psi(t))\Psi^{-1}(t)$$

Thus, matrix $D(t) = \Psi^{-1}(t) = (\Phi(t) - I_n)^{-1}$ satisfies the following linear differential equation

$$\dot{D}(t) = D(t)(A(t) - F(t)C(t)) - (A(t) - B(t)K(t))D(t) - F(t)C(t) \qquad (2.8)$$

with initial condition

$$D(0) = (\Phi_0 - I_n)^{-1}$$

In this case, the parameters of an equivalent regulator are

$$\tilde{z}_0 = (I_0 + D^{-1}(0))z_0 \ ,$$

$$\tilde{F}(t) = (I_n + D^{-1}(t))F(t), \ \tilde{K}(t) = K(t)(I_n + D(t))^{-1}D(t). \qquad (2.9)$$

Further on, we shall consider only optimal regulators (minimizing the quadratic functional (2.5)).

Regulator (2.3),(2.4) with parameters given by the separation theorem [1] is commonly used to control a system under incomplete information

$$z_0 = m_0,$$

$$F(t) = \Delta(t)C^T(t)V^{-1}(t), \ K(t) = N^{-1}(t)B^T(t)L(t) \qquad (2.10)$$

where matrix $\Delta(t) = E(x(t) - z(t))(x(t) - z(t))^T$ is a solution of differential equation

$$\dot{\Delta}(t) = A(t)\Delta(t) + \Delta(t)A^T(t) - \Delta(t)C^T(t)V^{-1}(t)C(t)\Delta(t) + S(t), \qquad (2.11)$$

with initial condition

$$\Delta(0) = \Delta_0$$

and matrix $L(t)$ is a solution of equation

$$\dot{L}(t) = -A^T(t)L(t) - L(t)A(t) + L(t)B(t)N^{-1}(t)B^T(t)L(t) - M(t), \tag{2.12}$$

with initial condition

$$L(T) = G$$

It follows from the consideration above that the separation theorem determines only one of possible optimal control realizations. Actually, a whole class of equivalent regulators may be described by theorem 1 for the regulator (2.10),(2.12). Obviously, all of such controllers are optimal.

3. Regulator with Random Noises in Dynamic Block

In this section, we consider in detail a class of optimal regulators $\tilde{U} = (\tilde{z}_0, \tilde{F}(t), \tilde{K}(t))$ that are equivalent to the separation theorem regulator (2.10)-(2.12) $U = (z_0, F(t), K(t))$. Matrices $\tilde{F}(t)$ and $\tilde{K}(t)$ are given by (2.7) where $\Phi(t)$ is a solution of Cauchy problem (2.6).

In practice, functioning of an optimal controller is accompanied by different distortions which are connected with analogous or digital simulation of the dynamic block. In present note , errors of all sorts are modeled by random noises in the dynamic block of the controller. Similar approach to the estimation problem was suggested in [2],[3].

Consider the following dynamic block

$$\dot{\tilde{z}}(t) = A(t)\tilde{z}(t) + B(t)\tilde{u}(t) + \tilde{F}(t)(\dot{y}(t) - C(t)\tilde{z}(t)) + \nu(t) \tag{3.1}$$

where $\{\nu(t)\}$ is white Gaussian, independent of other system noises random process with properties $E\{\nu(t)\} = 0$, $E\{\nu(t)\nu^T(\tau)\} = Q(t)\delta(t-\tau)$, $Q(t) \geq 0$.

We assume

$$\tilde{z}(0) = \Phi_0(m_0 + \theta) \tag{3.2}$$

where $m_0 = Ex_0$, and vector θ that models errors in determination of the dynamic block initial state, is given by $E\theta = 0$, $E\theta\theta^T = \Theta$, $\Theta \geq 0$.

Let feedback be formed by

$$u(t) = -K(t)\tilde{z}(t)$$

The state vector $r(t) = \begin{pmatrix} x(t) - \tilde{z}(t) \\ \tilde{z}(t) \end{pmatrix}$ of closed-loop dynamic system satisfies the next equation

$$\dot{r}(t) = \begin{bmatrix} A(t) - \tilde{F}(t)C(t) & 0 \\ \tilde{F}(t)C(t) & A(t) - B(t)\tilde{K}(t) \end{bmatrix} r(t) + \begin{bmatrix} I_n & -\tilde{F}(t) & -I_n \\ 0 & \tilde{F}(t) & I_n \end{bmatrix} \zeta(t), \tag{3.3}$$

with united noises vector

$$\zeta(t) = \begin{bmatrix} \xi(t) \\ \eta(t) \\ \nu(t) \end{bmatrix}$$

and initial condition

$$r(0) = \begin{pmatrix} x_0 - \tilde{z}_0 \\ \tilde{z}_0 \end{pmatrix}.$$

Then, we have for matrix $R(t) = E(r(t) - Er(t))(r(t) - Er(t))^T$

$$\dot{R}(t) = \mathcal{A}(t)R(t) + R(t)\mathcal{A}^T(t) + \Gamma(t) + \mathcal{Q}(t) \tag{3.4}$$

and initial condition

$$R(0) = \begin{bmatrix} \Delta_0 + \Phi_0\Theta\Phi_0^T & -\Phi_0\Theta\Phi_0^T \\ -\Phi_0\Theta\Phi_0^T & \Phi_0\Theta\Phi_0^T \end{bmatrix}$$

Here $\mathcal{A}(t), \Gamma(t)$ and $\mathcal{Q}(t)$ are partitioned according to

$$\mathcal{A}(t) = \begin{bmatrix} A(t) - \tilde{F}(t)C(t) & 0 \\ \tilde{F}(t)C(t) & A(t) - B(t)\tilde{K}(t) \end{bmatrix}, \ \mathcal{Q}(t) = \begin{bmatrix} Q(t) & -Q(t) \\ -Q(t) & Q(t) \end{bmatrix}.$$

$$\Gamma(t) = \begin{bmatrix} \tilde{F}(t)V(t)\tilde{F}(t) + S(t) & -\tilde{F}(t)V(t)\tilde{F}(t) \\ -\tilde{F}(t)V(t)\tilde{F}(t) & \tilde{F}(t)V(t)\tilde{F}(t) \end{bmatrix}.$$

In such circumstances criterion (2.5) may by rewritten in such a way

$$J = \int_o^T \mathrm{tr}(\mathcal{M}(t)R(t))dt + \mathrm{tr}(\mathcal{G}R(T)) \tag{3.5}$$

where

$$\mathcal{M}(t) = \begin{bmatrix} M(t) & M(t) \\ M(t) & \tilde{K}^T(t)N(t)\tilde{K}(t) + M(t) \end{bmatrix}, \ \mathcal{G} = \begin{bmatrix} G & G \\ G & G \end{bmatrix},$$

$R(t)$ is a solution of system (3.4).

Since all system noises are independent, it is easy to prove the following presentation of functional (3.5)

$$J = J_0 + J_\theta + J_\nu \tag{3.6}$$

where

$$J_0 = \int_o^T \mathrm{tr}(\mathcal{M}(t)R_0(t))dt + \mathrm{tr}(\mathcal{G}R_0(T))$$

is the optimal value of criterion (2.5) in the case of $\nu(t) \equiv 0, \theta = 0$ and matrix $R_0(t)$ satisfies

$$\dot{R}_0(t) = \mathcal{A}(t)R_0(t) + \mathcal{A}^T(t) + \Gamma(t) \tag{3.7}$$

with condition

$$R_0(0) = \begin{bmatrix} \Delta_0 & 0 \\ 0 & 0 \end{bmatrix}$$

The integral

$$J_\theta = \int_o^T \mathrm{tr}(\mathcal{M}(t)R_\theta(t))dt$$

is a term caused by the influence of indeterminacy of initial value $\tilde{z}_0$ $(\nu(t) \equiv 0, \theta \neq 0)$ and matrix $R_\theta(t)$ is given by

$$\dot{R}_\theta(t) = \mathcal{A}(t)R_\theta(t) + R_\theta(t)\mathcal{A}^T(t) \tag{3.8}$$

with condition

$$R_\theta(0) = \begin{bmatrix} \Phi_0\Theta\Phi_0^T & -\Phi_0\Theta\Phi_0^T \\ -\Phi_0\Theta\Phi_0^T & \Phi_0\Theta\Phi_0^T \end{bmatrix}$$

The integral

$$J_\nu = \int_o^T \mathrm{tr}(\mathcal{M}(t)R_\nu(t))dt$$

is a term caused by additional noise in the dynamic block (3.1) ($\nu(t) \neq 0, \theta = 0$) and matrix $R_\nu(t)$ satisfies

$$\dot{R}_\nu(t) = \mathcal{A}(t)R_\nu(t) + R_\nu(t)\mathcal{A}^T(t) + \mathcal{Q}(t), \tag{3.9}$$

with condition

$$R_\nu(0) = 0$$

With respect to the *Note* 1 of *Theorem* 1, J_0 has the same values for all regulators that are equivalent to the separation theorem regulator (2.10)-(2.12). Meanwhile, terms J_θ and J_ν have different values for different "equivalent" regulators. So, they depend on matrix Φ_0.

Let consider the following example

$$\begin{array}{l} \dot{x}(t) = u(t) + \xi(t), \quad t \in [0,1] \\ \dot{y} = x(t) + \eta(t), \end{array} \tag{3.10}$$

$$E\xi(t)\xi^T(\tau) = s\delta(t-\tau), \ E\eta(t)\eta^T(\tau) = v\delta(t-\tau)$$

where

$$s = 0.01, \ v = M = N = 1, \ G = 0, \ \Delta_0 = 0, \ \Theta = 0.005, \ q = 0.01$$

Under such conditions we have $J_0 = 0.005, J = 0.0075$ for the separation theorem regulator ($\varphi_0 = 1$) and $J = 0.0066$ for one of the "equivalent" regulators ($\varphi_0 = 150$).

Evidently, if there are additional noises ($\Theta \neq 0, Q \neq 0$) then the separation theorem regulator is not the best one in the equivalent class. Thus, the problem arises to choose a parameter Φ_0 that minimizes the quadratic criterion.

To simplify the following discussion, we consider the scalar case

$$\dot{x}(t) = a(t)x(t) + b(t)u(t) + \xi(t), \ t \in [0,T]$$

$$\dot{y}(t) = c(t)x(t) + \eta(t),$$

$$\dot{z}(t) = a(t)z(t) + b(t)u(t) + f(t)(\dot{y}(t) - c(t)z(t)) + \nu(t)$$

$$u(t) = -k(t)z(t)$$

$$E\xi(t)\xi^T(\tau) = s(t)\delta(t-\tau), \ E\eta(t)\eta^T(\tau) = v(t)\delta(t-\tau), \ E\nu(t)\nu^T(\tau) = q(t)\delta(t-\tau)$$

and , since J_0 does not depend on φ_0 , we confine to optimization of $\bar{J} = J_\theta + J_\nu$.

It may be easily shown that

$$\bar{J}(\varphi_0) = \int_o^T \mathrm{tr}(\mathcal{M}(t)\bar{R}(t))dt$$

where

$$\dot{\bar{R}}(t) = \mathcal{A}(t)\bar{R}(t) + \bar{R}(t)\mathcal{A}^T(t) + \mathcal{Q}(t) \tag{3.11}$$

$$\bar{R}(0) = \begin{bmatrix} \varphi_0^2\Theta & -\varphi_0^2\Theta \\ -\varphi_0^2\Theta & \varphi_0^2\Theta \end{bmatrix}$$

Let $\varphi(t)$ and $\bar{r}_{11}(t), \bar{r}_{12}(t), \bar{r}_{22}(t)$ (the corresponding elements of matrix $\bar{R}(t)$) be designated as

$$\alpha_1 = \varphi(t),\ \alpha_2 = \bar{r}_{11}(t),\ \alpha_3 = \bar{r}_{12}(t),\ \alpha_4 = \bar{r}_{22}(t).$$

Then the relations (2.6),(3.11) may by rewritten as a system of differential equations

$$\dot{\alpha}_i = \psi_i(t, \alpha_1, \alpha_2, \alpha_3, \alpha_4),\ i = 1, 2, 3, 4, \tag{3.12}$$

with conditions

$$\alpha_1(0) = \varphi_0,\ \alpha_2(0) = \varphi_0^2\Theta,\ \alpha_3(0) = -\varphi_0^2\Theta,\ \alpha_4 = \varphi_0^2\Theta$$

where

$$\psi_1(t, \alpha_1) = \alpha_1(f(t)c(t) + b(t)k(t)) - \alpha_1^2 f(t)c(t) - b(t)k(t)$$

$$\psi_2(t, \alpha_1, \alpha_2) = 2\alpha_2(a(t) - \alpha_1 f(t)c(t)) + q(t)$$

$$\psi_3(t, \alpha_1, \alpha_2, \alpha_3) = \alpha_1\alpha_2 f(t)c(t) + \alpha_3(2a(t) - \alpha_1 f(t)c(t) - b(t)k(t)/\alpha_1) - q(t)$$

$$\psi_4(t, \alpha_1, \alpha_3, \alpha_4) = 2\alpha_1\alpha_3 f(t)c(t) + 2\alpha_4(a(t) - b(t)k(t)/\alpha_1) + q(t)$$

In this case functional $\bar{J}(\varphi_0)$ has the following form

$$\bar{J}(\varphi_0) = \int_0^T \Omega(t, \alpha_1, \alpha_2, \alpha_3, \alpha_4)dt \tag{3.13}$$

where

$$\Omega(t, \alpha_1, \alpha_2, \alpha_3, \alpha_4) = M(t)(\alpha_2 + 2\alpha_3 + \alpha_4) + k^2(t)N(t)\alpha_4/\alpha_1^2$$

The necessary condition of extremum $\bar{J}'_{\varphi_0} = 0$ is presented by the integral equation

$$\int_0^T (\frac{\partial\Omega}{\partial\alpha_1}\beta_1(t) + \frac{\partial\Omega}{\partial\alpha_2}\beta_2(t) + \frac{\partial\Omega}{\partial\alpha_3}\beta_3(t) + \frac{\partial\Omega}{\partial\alpha_4}\beta_4(t))dt = 0 \tag{3.14}$$

where

$$\beta_i(t) = \frac{\partial\psi_i}{\partial\varphi_0},\ i = 1, 2, 3, 4$$

satisfy the system in variations

$$\dot{\beta}_i(t) = \sum_{j=1}^{4} \frac{\partial\psi_i}{\partial\alpha_j}\beta_j(t),\ \beta_i(0) = \frac{\partial\alpha_i(0)}{\partial\varphi_0}.$$

With respect to the equations (3.12), derive the following differential system for $\beta_i(t)$

$$\dot{\beta}_1(t) = (f(t)c(t) + b(t)k(t) - 2\alpha_1(t)f(t)c(t))\beta_1(t)$$

$$\dot{\beta}_2(t) = -2\alpha_2(t)f(t)c(t)\beta_1(t) + 2(a(t) - \alpha_1(t)f(t)c(t))\beta_2(t)$$

$$\dot{\beta}_3(t) = (f(t)c(t)(\alpha_2(t) - \alpha_3(t))) + \alpha_3(t)b(t)k(t)/\alpha_1^2(t))\beta_1(t) + \tag{3.15}$$

$$+\alpha_1(t)f(t)c(t)\beta_2(t) + (2a(t) - \alpha_1(t)f(t)c(t) - b(t)k(t)/\alpha_1(t))\beta_3(t)$$

$$\dot{\beta}_4(t) = 2(\alpha_3(t)f(t)c(t) + \alpha_4(t)b(t)k(t)/\alpha_1^2(t))\beta_1(t) + 2\alpha_1(t)f(t)c(t)\beta_3(t) +$$

$$+2(a(t) - b(t)k(t)/\alpha_1(t))\beta_4(t)$$

with conditions

$$\beta_1(0) = 1,\ \beta_2(0) = 2\varphi_0\Theta,\ \beta_3(0) = -2\varphi_0\Theta,\ \beta_4(0) = 2\varphi_0\Theta$$

Finally, the necessary condition of extremum is (see (3.13),(3.14))

$$\int_0^T \left[(\beta_4(t)\alpha_1(t) - 2\alpha_4(t)\beta_1(t)) \frac{k^2(t)N(t)}{\alpha_1^3(t)} + M(t)(\beta_2(t) + 2\beta_3(t) + \beta_4(t)) \right] dt = 0 \tag{3.16}$$

Here $\alpha_i(t), \beta_i(t)$ satisfy accordingly the systems (3.12),(3.15) and $f(t), k(t)$ are the parameters of the separation theorem regulator (2.10),(2.12). Thus, to solve the problem we must find the parameter φ_0 (see (3.12),(3.15)) for which the equation (3.16) is carried out.

We have designed the regulator that is the least sensitive to additional noises. It is optimal only in the class of regulators that are equivalent to the separation theorem regulator. The design of an optimal controller with taking into account the presence of additional noises is examined in the section 4.

4. Optimal regulator

In this section we consider a linear stochastic system (2.1),(2.2) with regulator which consists of dynamic block

$$\dot{z}(t) = A(t)z(t) + B(t)u(t) + F(t)(\dot{y}(t) - C(t)z(t)) + \nu(t) \tag{4.1}$$

with initial state

$$z(o) = m_0$$

and feedback

$$u = -K(t)z(t) \tag{4.2}$$

We assume that initial state of dynamic block (4.1) is fixed. Therefore, cost performance $J[u]$ from (2.5) depends on the set parameters $U = (F(t), K(t))$ only. Consider the following optimal control problem

$$J[u] \to \inf_U \tag{4.3}$$

Theorem 2: The optimal parameters $F(t), K(t)$ of control problem (4.3) satisfy the next equalities

$$\dot{R}_{11}(t)=(A(t)-F(t)C(t))R_{11}(t)+R_{11}(t)(A(t)-F(t)C(t))^T+$$
$$+F(t)V(t)F^T(t)+S(t)+Q(t)$$
$$\dot{R}_{12}(t)=(A(t)-F(t)C(t))R_{12}(t)+R_{12}(t)(A(t)-B(t)K(t))^T+ \tag{4.4}$$
$$+R_{11}(t)C^T(t)F^T(t)-F(t)V(t)F^T(t)-Q(t)$$

$$\dot{R}_{22}(t)=(A(t)-B(t)K(t))R_{22}(t)+R_{22}(t)(A(t)-B(t)K(t))^T+$$

$$+R_{12}^T(t)C^T(t)F^T(t)+F(t)C(t)R_{12}(t)+F(t)V(t)F^T(t)+Q(t)$$

$$R_{11}(0)=\Delta_{0,}\ R_{12}(0)=0,\ R_{22}(0)=0$$

$$\dot{W}_{11}(t)+(A(t)-F(t)C(t))^TW_{11}(t)+W_{11}(t)(A(t)-F(t)C(t))+$$
$$+W_{12}(t)F(t)C(t)+C^T(t)F^T(t)W_{12}^T(t)+M(t)=0$$

$$\dot{W}_{12}(t)+(A(t)-F(t)C(t))^TW_{12}(t)+W_{12}(t)(A(t)-B(t)K(t))+ \tag{4.5}$$

$$+C^T(t)F^T(t)W_{22}(t)+M(t)=0$$
$$\dot{W}_{22}(t)+(A(t)-B(t)K(t))^TW_{22}(t)+W_{22}(t)(A(t)-B(t)K(t))+$$
$$+K^T(t)N(t)K(t)+M(t)=0$$
$$W_{11}(T)=G,\ W_{12}(T)=G,\ W_{22}(T)=G$$

$$V(t)F^T(t)(W_{11}(t)-W_{12}(t)-W_{12}^T(t)+W_{22}(t))= \tag{4.6}$$

$$=C(t)(R_{11}(t)(W_{11}(t)-W_{12}(t))+R_{12}(t)(W_{12}^T(t)-W_{22}(t)))$$

$$R_{22}(t)K^T(t)N(t)=(R_{12}^T(t)W_{12}(t)+R_{22}(t)W_{22}(t))B(t) \tag{4.7}$$

Proof: Let

$$e(t)=x(t)-z(t),\ r(t)=\begin{bmatrix} e(t) \\ z(t) \end{bmatrix},\ R(t)=\begin{bmatrix} R_{11}(t) & R_{12}(t) \\ R_{21}(t) & R_{22}(t) \end{bmatrix}$$

where

$$R_{11}(t)=E(e(t)-Ee(t))(e(t)-Ee(t))^T, R_{12}(t)=E(e(t)-Ee(t))(z(t)-Ez(t))^T,$$
$$R_{21}(t)=R_{12}^T(t), R_{22}(t)=E(z(t)-Ez(t))(z(t)-Ez(t))^T$$

The matrix $R(t)=E(r(t)-Er(t))(r(t)-Er(t))^T$ satisfies the differential equation

$$\dot{R}(t)=\mathcal{A}(t)R(t)+R(t)\mathcal{A}^T(t)+\Gamma(t)+\mathcal{Q}(t) \tag{4.8}$$

and initial condition

$$R(0) = R_0 = \begin{bmatrix} \Delta_0 & 0 \\ 0 & 0 \end{bmatrix} \tag{4.9}$$

where

$$\mathcal{A}(t) = \begin{bmatrix} A(t) - F(t)C(t) & 0 \\ F(t)C(t) & A(t) - B(t)K(t) \end{bmatrix}$$

$$\Gamma(t) = \begin{bmatrix} F(t)V(t)F^T(t) + S(t) & -F(t)V(t)F^T(t) \\ -F(t)V(t)F^T(t) & F(t)V(t)F^T(t) \end{bmatrix}, \ \mathcal{Q}(t) = \begin{bmatrix} Q(t) & -Q(t) \\ -Q(t) & Q(t) \end{bmatrix}$$

Criterion (2.5) has a form

$$J = \int_0^T \mathrm{tr}(\mathcal{M}(t)R(t))dt + \mathrm{tr}(\mathcal{G}R(T)) \tag{4.10}$$

where

$$\mathcal{M}(t) = \begin{bmatrix} M(t) & M(t) \\ M(t) & K(t)^T N(t)K(t) + M(t) \end{bmatrix}, \ \mathcal{G} = \begin{bmatrix} G & G \\ G & G \end{bmatrix}$$

Thus, stochastic problem (4.1)-(4.3) may be rewritten as a deterministic problem to choose matrices $F(t)$ and $K(t)$ which minimize the functional (4.10) with restrictions (4.8),(4.9). The similar approach was used in [6],[7].

Lagrange function for this variational problem is

$$\mathcal{H} \ = \mathcal{H}(R, W, F, K) = \mathrm{tr}((\mathcal{A}(t)R(t) + R(t)\mathcal{A}^T(t) + \Gamma(t) + \mathcal{Q}(t))W(t)) + \mathrm{tr}(\mathcal{M}(t)R(t))$$

where $W = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix}$ is a matrix of Lagrange multipliers.

The necessary conditions of extremum are presented by the system

$$\dot{R} = \frac{\partial \mathcal{H}}{\partial W}, \ R(0) = R_0 \tag{4.11}$$

$$\dot{W} = -\frac{\partial \mathcal{H}}{\partial R}, \ W(T) = \mathcal{G} \tag{4.12}$$

$$\frac{\partial \mathcal{H}}{\partial F} = 0, \ \frac{\partial \mathcal{H}}{\partial K} = 0 \tag{4.13}$$

Using the relations

$$X = \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix}, \ Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

$$\mathrm{tr}(X + Y) = \mathrm{tr}(X) + \mathrm{tr}(Y)$$

$$\mathrm{tr}(XY) = \mathrm{tr}(YX) = \mathrm{tr}(X_{11}Y_{11} + X_{12}Y_{21} + X_{21}Y_{12} + X_{22}Y_{22})$$

we have the next representations

$$\mathrm{tr}((\mathcal{A}R + R\mathcal{A}^T)W) = \mathrm{tr}((\mathcal{A}^T W + W\mathcal{A})R)$$

$$\frac{\partial \mathcal{H}}{\partial W} = \mathcal{A}R + R\mathcal{A}^T + \Gamma + \mathcal{Q} \tag{4.14}$$

$$\frac{\partial \mathcal{H}}{\partial R} = \mathcal{A}^T W + W\mathcal{A} + \mathcal{M} \quad (4.15)$$

$$\frac{\partial \mathcal{H}}{\partial F} = -2C(R_{11}(W_{11} - W_{12}) + R_{12}(W_{12}^T - W_{22}))+ \quad (4.16)$$

$$+2VF^T(W_{11} - W_{12} - W_{12}^T + W_{22})$$

$$\frac{\partial \mathcal{H}}{\partial K} = -2(R_{12}^T W_{12} + R_{22}W_{22})B + 2R_{22}K^T N \quad (4.17)$$

It follows from (4.14)-(4.17) that necessary conditions (4.11)-(4.13) can be written as system

$$\dot{R} = \mathcal{A}R + R\mathcal{A}^T + \Gamma + \mathcal{Q},\ R(0) = R_0$$

$$\dot{W} = \mathcal{A}^T W + W\mathcal{A} + \mathcal{M},\ W(T) = \mathcal{G}$$

$$-C(R_{11}(W_{11} - W_{12}) + R_{12}(W_{12}^T - W_{22}))+$$
$$+VF^T(W_{11} - W_{12} - W_{12}^T + W_{22}) = 0$$

$$-(R_{12}^T W_{12} + R_{22}W_{22})B + R_{22}K^T N = 0$$

Using the block representation of matrices $R, \mathcal{A}, \Gamma, \mathcal{Q}, W, \mathcal{M}, \mathcal{G}$ we have equations (4.4)-(4.7).

5. Time-Invariant Systems

Consider a time-invariant system

$$\dot{x}(t) = Ax(t) + Bu(t) + \xi(t), \quad (5.1)$$
$$\dot{y}(t) = Cx(t) + \eta(t) \quad (5.2)$$

where

$$E\{\xi(t)\} = 0,\ E\{\xi(t)\xi(\tau)^T\} = S\delta(t-\tau),\ S \geq 0$$

$$E\{\eta(t)\} = 0,\ E\{\eta(t)\eta(\tau)^T\} = V\delta(t-\tau),\ V > 0$$

To control the system (5.1) let design a regulator that consists of dynamic block

$$\dot{z}(t) = Az(t) + Bu(t) + F(\dot{y}(t) - Cz(t)) \quad (5.3)$$

and feedback

$$u(t) = -Fz(t). \quad (5.4)$$

In this case the quadratic criterion is

$$J[u] = \lim_{t\to\infty} E[x^T(t)Mx(t) + u^T(t)Nu(t)], M \geq 0, N > 0 \tag{5.5}$$

We assume that (A, B) is stabilized and (A, C) is detectable. Then, sets $\mathcal{F} = \{F \mid A - FC-\text{stable}\}$ and $\mathcal{K} = \{K \mid A - BK-\text{stable}\}$ are not empty. Regulator (5.3),(5.4) with parameters $U = (F, K)(F \in \mathcal{F}, K \in \mathcal{K})$ will be called stabilizing. For any of such regulators a closed-loop system

$$\dot{\lambda} = \mathcal{A}_1\lambda + \mu \tag{5.6}$$

where

$$\lambda(t) = \begin{pmatrix} x(t) \\ z(t) \end{pmatrix}, \ \mu(t) = \begin{pmatrix} \xi(t) \\ F\eta(t) \end{pmatrix}, \ \mathcal{A}_1 = \begin{bmatrix} A & -BK \\ FC & A - BK - FC \end{bmatrix},$$

and control signal (5.4) have stationary distributed states $\lambda = \begin{pmatrix} x \\ z \end{pmatrix}, u = -Kz$ that are independent of x_0, z_0. The second moment matrix $\Lambda = E\lambda\lambda^T$ of stationary distributed state λ satisfies the equation

$$\mathcal{A}_1\Lambda + \Lambda\mathcal{A}_1^T + \Gamma_1 = 0 \tag{5.7}$$

where

$$\Gamma_1 = \begin{bmatrix} S & 0 \\ 0 & FVF^T \end{bmatrix}.$$

In this circumstances functional (5.5) is

$$J[u] = E[x^TMx + u^TNu] = \mathrm{tr}(M\Lambda_{11} + K^TNK\Lambda_{22}), \tag{5.8}$$

Here $\Lambda_{11}, \Lambda_{22}$ are the corresponding parts of matrix Λ

$$\Lambda = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{bmatrix}, \ \Lambda_{21} = \Lambda_{12}^T$$

Consider a stabilizing regulator $U = (F, K)$ and some regulator $\tilde{U} = (\tilde{F}, \tilde{K})$, and prove the following result.

Theorem 3: Let the parameters of regulator $\tilde{U}$ be given by

$$\tilde{F} = \Phi F, \ \tilde{K} = K\Phi^{-1} \tag{5.9}$$

where non-singular matrix Φ satisfies the equation

$$A\Phi + \Phi(FC + BK - A) - \Phi FC\Phi - BK = 0. \tag{5.10}$$

Then

1) regulator $\tilde{U}$ is also stabilizing

$$\tilde{F} \in \mathcal{F}, \ \tilde{K} \in \mathcal{K}; \tag{5.11}$$

2) the following correlation is carried out

$$Exx^T = E\tilde{x}\tilde{x}^T, \ Euu^T = E\tilde{u}\tilde{u}^T \tag{5.12}$$

for stationary distributed states $\lambda = \begin{pmatrix} x \\ z \end{pmatrix}$, $\tilde{\lambda} = \begin{pmatrix} \tilde{x} \\ \tilde{z} \end{pmatrix}$ and stationary distributed controls $u = -Kz$, $\tilde{u} = -\tilde{K}\tilde{z}$ that correspond to regulators U, $\tilde{U}$.

Proof: Consider matrices

$$\mathcal{A} = \begin{bmatrix} A - FC & 0 \\ FC & A - BK \end{bmatrix}, \ \mathcal{A}_1 = \begin{bmatrix} A - \tilde{F}C & 0 \\ \tilde{F}C & A - B\tilde{K} \end{bmatrix}, \ \mathcal{D} = \begin{bmatrix} I_n & I_n - \Phi \\ 0 & I_n \end{bmatrix}.$$

With respect to (5.9),(5.10) , we have $\mathcal{A} = \mathcal{D}^{-1}\mathcal{A}_1\mathcal{D}$, i.e. matrices $\mathcal{A}$ and $\mathcal{A}_1$ are similar. Formula (5.11) follows directly from the coincidence of the similar matrices spectra. Matrix $\tilde{\Lambda} = E\tilde{\lambda}\tilde{\lambda}^T$ satisfies the equation

$$\mathcal{A}_2\tilde{\Lambda} + \tilde{\Lambda}\mathcal{A}_2^T + \mathcal{S} = 0$$

where

$$\mathcal{A}_2 = \begin{bmatrix} A & -B\tilde{K} \\ \tilde{F}C & A - B\tilde{K} - \tilde{F}C \end{bmatrix}, \ \mathcal{S} = \begin{bmatrix} S & 0 \\ 0 & \tilde{F}V\tilde{F}^T \end{bmatrix}.$$

Using (5.9),(5.10), it is easily shown that Λ from (5.7) and $\tilde{\Lambda}$ are connected by

$$\tilde{\Lambda} = \mathcal{S}_1\Lambda\mathcal{S}_1^T, \ \mathcal{S}_1 = \begin{bmatrix} I_n & 0 \\ 0 & \Phi \end{bmatrix}$$

Equalities (5.12) come directly from the following relations

$$Exx^T = \Lambda_{11}, \ E\tilde{x}\tilde{x}^T = \tilde{\Lambda}_{11},$$

$$Euu^T = K\Lambda_{22}K^T, \ E\tilde{u}\tilde{u}^T = \tilde{K}\tilde{\Lambda}_{22}\tilde{K}^T$$
$$\tilde{\Lambda}_{22} = \Phi\Lambda_{22}\Phi^T, \ \tilde{K} = K\Phi^{-1}.$$

Theorem is proved.

Note, the quadratic criterion (5.8) has the same values for all the controls $\tilde{u}$ that are formed by the regulators $\tilde{U}$ of the class described, $J[\tilde{u}] = J[u]$. Here parameters $\tilde{F}$ and $\tilde{K}$are given by (5.9) when Φ is a solution of the equation (5.10).

Evidently, the variety of regulators $\tilde{U}$ that are equivalent to a starting regulator U, depends on the number of solutions (5.10). The maximum number equals to C_{2n}^n. The case with only two solutions is of special interest. Assume that matrix $\Phi - I_n$ is non-singular and designate $D = (\Phi - I_n)^{-1}$. Then D satisfies the following linear equation

$$D(A - FC) - (A - BK)D - FC = 0 \tag{5.13}$$

If spectra of matrices $A - FC$ and $A - BK$ don't intersect then this equation has one solution. Under such circumstances, there are only two equivalent regulators: a starting controller $U = (F, K)$ and the alternative one $\tilde{U} = ((D^{-1} + I_n)F, K(D + I_n)^{-1}D)$.

Now, we take the regulator

$$\dot{z}(t) = Az(t) + Bu(t) + F(\dot{y}(t) - Cz(t)) + \nu(t)$$

$$u(t) = -Fz(t).$$

with additional noise $\nu(t)$ in the dynamic block :

$$E\{\nu(t)\} = 0, E\{\nu(t)\nu^T(\tau)\} = Q\delta(t-\tau), Q > 0$$

We consider a class of regulators $U = (F, K)$ which are equivalent to the separation theorem regulator $U_0 = (F_0, K_0)$

$$F_0 = \Delta_0 C^T V^{-1},\ K_0 = N^{-1}B^T L.$$

Here Δ_0 and L are given by

$$A\Delta_0 + \Delta_0 A^T - \Delta_0 C^T V^{-1} C\Delta_0 + S = 0$$

$$A^T L + LA - LBN^{-1}B^T L + M = 0$$

The quadratic functional (5.5) may be presented (see (3.6)-(3.9))

$$J = J_0 + J_\nu$$

where:

$$J_0 = \mathrm{tr}(\mathcal{M}R_0)$$

and matrix R_0 satisfies

$$\mathcal{A}R_0 + R_0\mathcal{A}^T + \Gamma = 0;$$

$$J_\nu = \mathrm{tr}(\mathcal{M}R_\nu)$$

and matrix R_ν satisfies

$$\mathcal{A}R_\nu + R_\nu\mathcal{A}^T + \mathcal{Q} = 0.$$

Matrices $\mathcal{A}, \Gamma, \mathcal{M}$ and $\mathcal{Q}$ have the following structure

$$\mathcal{A} = \begin{bmatrix} A - FC & 0 \\ FC & A - BK \end{bmatrix}, \Gamma = \begin{bmatrix} FVF^T + S & -FVF^T \\ -FVF^T & FVF^T \end{bmatrix},$$

$$\mathcal{M} = \begin{bmatrix} M & M \\ M & K^T NK + M \end{bmatrix}, \mathcal{Q} = \begin{bmatrix} Q & -Q \\ -Q & Q \end{bmatrix}$$

Here as in(3.6), J_0 is the optimal value of criterion when $\nu(t) \equiv 0$ which is the same for all "equivalent" regulators; J_ν is a term caused by the influence of disregarded additional disturbances and its values are different for different matrices Φ. The term J_θ is absent because stationary criterion doesn't depend on the initial information.

The examples show that the equivalent regulator U may be less receptive towards additional noises then the separation theorem regulator $U_{0.}$

Now, we derive the optimal regulator which takes into account the influence of additional noises.

Let $R = Err^T$ be the second moment matrix of stationary distributed state

$r = \begin{bmatrix} x - z \\ z \end{bmatrix}$ of a closed-loop system

$$\dot{r}(t) = \mathcal{A}r(t) + \beta(t)$$

where

$$r(t) = \begin{bmatrix} x(t) - z(t) \\ z(t) \end{bmatrix}, \ \beta(t) = \begin{bmatrix} \xi(t) - F\eta(t) - \nu(t) \\ F\eta(t) + \nu(t) \end{bmatrix}, \ F \in \mathcal{F}, \ K \in \mathcal{K}$$

The blocks of matrix $R = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix}$ are

$$R_{11} = E(x - z)(x - z)^T, R_{12} = E(x - z)z^T, R_{21} = R_{12}^T, R_{22} = Ezz^T$$

They satisfy the equations

$$(A - FC)R_{11} + R_{11}(A - FC)^T + FVF^T + S + Q = 0$$

$$(A - FC)R_{12} + R_{12}(A - BK)^T + R_{11}C^T F^T - FVF^T - Q = 0 \qquad (5.14)$$

$$(A - BK)R_{22} + R_{22}(A - BK)^T + R_{12}^T C^T F^T + FCR_{12} + FVF^T + Q = 0$$

Criterion (5.5) can be rewritten as

$$J = \mathrm{tr}(M(R_{11} + R_{12} + R_{12}^T + R_{22}) + K^T NKR_{22}) \qquad (5.15)$$

Thus, stochastic optimal problem may be rewritten as deterministic problem to choose matrices $F \in \mathcal{F}$ and $K \in \mathcal{K}$ which minimize the criterion (5.15) with restrictions (5.14). Blocks of the matrix $W = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix}$ of Lagrange multipliers and the optimal parameters F and K satisfy the equations

$$(A - FC)^T W_{11} + W_{11}(A - FC) + W_{12}FC + C^T F^T W_{12}^T + M = 0$$

$$(A - FC)^T W_{12} + W_{12}(A - BK) + C^T F^T W_{22} + M = 0 \qquad (5.16)$$

$$(A - BK)^T W_{22} + W_{22}(A - BK) + K^T NK + M = 0$$

$$VF^T(W_{11} - W_{12} - W_{12}^T + W_{22}) = C(R_{11}(W_{11} - W_{12}) + R_{12}(W_{12}^T - W_{22})) \qquad (5.17)$$

$$R_{22}K^T N = (R_{12}^T W_{12} + R_{22}W_{22})B \qquad (5.18)$$

So, the necessary conditions of extremum are presented by the system(5.14),(5.16)-(5.18).

Example. Let demonstrate the received results by the following example

$$\dot{x}(t) = u(t) + \xi(t), \quad E\xi(t)\xi^T(\tau) = s\delta(t-\tau),$$
$$\dot{y}(t) = x(t) + \eta(t), \quad E\eta(t)\eta^T(\tau) = v\delta(t-\tau)$$

Consider a regulator

$$\dot{z}(t) = u(t) + f(\dot{y}(t) - z(t)) + \nu(t), E\nu(t)\nu^T(\tau) = q\delta(t-\tau),$$

$$u(t) = -kz(t)$$

In this case $\mathcal{F} = \{f \mid f > 0\}, \mathcal{K} = \{k \mid k > 0\}$ and a class of optimal (when $q = 0$) controllers consist of

the separation theorem regulator

$$f_0 = \sqrt{\frac{s}{v}}, \; k_0 = \sqrt{\frac{M}{N}}$$

and the alternative one

$$\tilde{f} = \sqrt{\frac{M}{N}}, \; \tilde{k} = \sqrt{\frac{s}{v}}$$

Here

$$J_0 = \tilde{J} = M\sqrt{sv} + s\sqrt{MN}$$

and additions caused by disturbances $\nu(t)$ are given by

$$J_\nu = \frac{q}{2}M\sqrt{\frac{v}{s}}, \; \tilde{J}_\nu = \frac{q}{2}N\sqrt{\frac{s}{v}} \tag{5.19}$$

The question of what regulator is better when $q \neq 0$, is solved by the relation of Mv and Ns . If $Mv > Ns$ as, for example, in the case of minimizing Ex^2 (M is large, N is small) or in the case of vanishing process noise ($s \to 0$) then the regulator $\tilde{U} = (\tilde{f}, \tilde{k})$ must be preferred. If $Mv < Ns$ (for example, v is small) then it is better to use the separation theorem regulator $U_0 = (f_0, k_0)$. At last, in the case of $Mv = Ns$ both regulators yield the same result $J_\nu = \tilde{J}_\nu$.

6. Conclusions

In this work, the problem of controlling linear stochastic systems under incomplete information is considered. A conception of the equivalent regulator is introduced and the constructive description of such regulator class is suggested. All equivalent regulator form control signals of the same value. The influence of additional noises in the dynamic block on the "optimal"regulators (equivalent to the separation theorem one) is investigated. It turns out that sometimes it may be much better to use a regulator that doesn't satisfy the separation principle.

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